Magnetic Forces Doing Work?

EUGENE P. MOSCA
Department of Physics
University of Vermont
Burlington, Vermont
(Received 4 September 1973; revised 11 October 1973)

Consider a conducting circuit moving with velocity $\mathbf{v}$ *through a constant magnetic field* $\mathbf{B}$. *The induced emf is given by* $\int \mathbf{v} \times \mathbf{B} \cdot d\mathbf{l}$ *where the integral is taken once around the circuit. Some texts refer to* $\mathbf{v} \times \mathbf{B}$ *as the force the magnetic field exerts on a unit charge moving around the circuit. This is incorrect as magnetic forces can never do work. The force the conductor exerts on an electron is shown to do this work.*

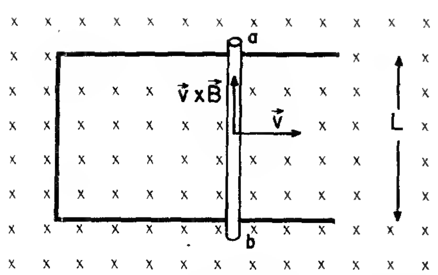

FIG. 1. The force per unit charge in a conducting rod moving with constant velocity through a magnetic field directed into the page.

A beginning physics student learns that the force a static magnetic field exerts on a charged particle always is directed perpendicular to the velocity of the charged particle. This means the force exerted by the magnetic field on the charged particle can never do any work on the particle. In spite of this, many physics texts solve for the emf along a conductor moving through a static magnetic field by implying the magnetic force does the work on the electrons as they move along. The arguments typically use a stationary U-shaped conductor with a conducting rod of length L sliding along it at constant velocity $\mathbf{v}$ (Fig. 1). This circuit is in a plane perpendicular to a uniform constant magnetic field. (You may consider this circuit to be placed between the jaws of a large, permanent magnet.)

The force per unit charge within the moving rod is given by $\mathbf{v} \times \mathbf{B}$. This force is tangential to the rod and does work vBL on a unit charge moving from b to a. This work is the emf in the circuit.

Up to this point the nature or origin of the force $\mathbf{v} \times \mathbf{B}$ has not been discussed. Many undergraduate physics texts, however, refer to $\mathbf{v} \times \mathbf{B}$ as a magnetic force. Such a reference implies that a magnetic force does work on a charge traversing the moving rod. This of course does not happen. (If it did happen the magnet would be supplying energy to the circuit. The magnet, since it would then be losing energy, would become colder. This is not observed.) As will be shown, the force $\mathbf{v} \times \mathbf{B}$ is the sum of two forces, the force the rod exerts on the charge and the magnetic force.

In the same circuit as before consider the forces on an electron traversing the rod (Fig. 2). The velocity $\mathbf{v}_e$ of an average electron makes an angle θ with the rod. The magnetic field exerts a force $\mathbf{F}_m$ and the rod exerts a force perpendicular to the rod $\mathbf{F}_r$. The force $\mathbf{F}$ is the resultant of these forces.

That is,

$$\mathbf{F} = \mathbf{F}_m + \mathbf{F}_r. \qquad (1)$$

The component of $\mathbf{v}_e$ to the right must equal the

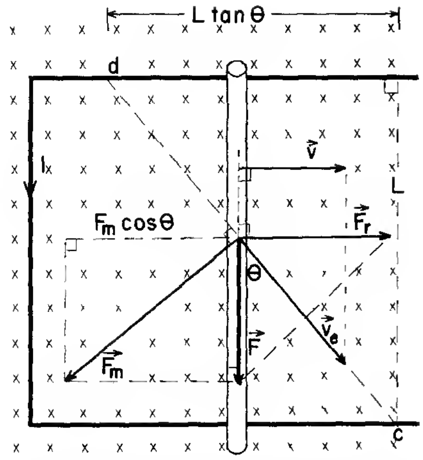

FIG. 2. A diagram of the forces on an electron traversing a conducting rod sliding along a U-shaped conductor. There is a constant, uniform magnetic field into the page.

speed of the rod. Thus,

$$v_e \sin\theta = v. \tag{2}$$

Because the electron moves to the right with constant speed the component of $\mathbf{F}_m$ to the left equals the force of the rod to the right. So,

$$F_m \cos\theta = F_r. \tag{3}$$

From this one sees that $\mathbf{F}$ is directed tangential to the wire as shown.

As the electron moves from d to c the force $\mathbf{F}$ acts through a distance L and does work on the electron equal to FL. From Fig. 2, one sees that

$$F = F_m \sin\theta. \tag{4}$$

From the Lorentz force law we have

$$F_m = ev_eB. \tag{5}$$

By substituting Eq. (5) into Eq. (4), one obtains

$$F = ev_eB \sin\theta. \tag{6}$$

Then substituting from Eq. (2) into Eq. (6), one sees

$$F = evB. \tag{7}$$

Or, in vector notation,

$$\mathbf{F} = -e\mathbf{v} \times \mathbf{B}. \tag{8}$$

This shows the force per unit charge within the wire is given by $\mathbf{v} \times \mathbf{B}$. The work done by $\mathbf{F}$ on the electron is given by $evBL$.

It is sometimes convenient to think of this work as being done by the force the rod exerts on the electron. From Fig. 2 one sees that the force $\mathbf{F}_r$ acts through a distance $L \tan\theta$. The work done by $\mathbf{F}_r$ equals $F_rL \tan\theta$.

By substituting from Eqs. (3) and (5), we see

$$(\text{Work done by } \mathbf{F}_r) = ev_eB \cos\theta \cdot L \tan\theta, \tag{9}$$

$$(\text{Work done by } \mathbf{F}_r) = ev_eBL \sin\theta. \tag{10}$$

Then substituting Eq. (2) into Eq. (10), one obtains,

$$(\text{Work done by } \mathbf{F}_r) = evBL.$$

This result confirms that the work done by $\mathbf{F}$ equals the work done by $\mathbf{F}_r$.

Because the rod exerts a force to the right on the electron, the electron exerts an equal force to the left on the rod. In order to keep the rod moving with constant speed an external agent must exert a force to the right on the rod. This agent is the one supplying the energy to the circuit.

A more complete picture of the energetics of the circuit can be found by considering all of the forces acting on a conduction electron traversing the rod. In addition to the magnetic force $\mathbf{F}_m$ and the force exerted by the rod $\mathbf{F}_r$ there is a dissipative resistive force, a magnetic force due to the current in the circuit, and an electric force due to the accumulation of charges at various parts of the circuit. In the steady state situation these forces cancel so that the net force on the average conduction electron is zero. The energy added to the electron by the force exerted on it by the rod $\mathbf{F}_r$ is transformed into thermal energy within the rod and into an increase of potential energy of

the electron. Because the U-shaped conductor has a finite resistance, charges are accumulated at the regions of contact with the moving rod. These charges exert an electric force on the electrons within the rod. It is the work done against this force that determines the difference in potential across the rod.

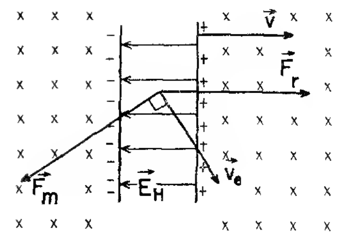

FIG. 3. Forces on an electron traversing a rod moving with speed v through a constant magnetic field **B**.

APPENDIX

The Nature of the Force the Conducting Rod, Moving through a Magnetostatic Field, Exerts on the Conduction Electrons

Figure 3 is a close up view of the conducting rod in Fig. 2. The component of $\mathbf{F}_m$ to the left causes the conduction electrons to drift to the left side of the rod, leaving the right side of the rod positively charged. These surface charges create an electric field $\mathbf{E}_H$ within the rod to the left. This is known as the Hall effect. The force of the rod on the electron is exerted via this electric field, i.e., $\mathbf{F}_r = -e\mathbf{E}_H$. The surface charges increase until the force $\mathbf{F}_r$ to the right balances the component of the magnetic force to the left.